

UITP-09/06
hep-th/0606474

Transplanckian signatures in WMAP3?

Ulf H. Danielsson

Institutionen för Teoretisk Fysik, Uppsala Universitet,
Box 803, SE-751 08 Uppsala, Sweden ulf@teorfys.uu.se

Abstract

In this note we investigate how a possible signal in the WMAP3 data of rapid oscillations in the primordial spectrum can be accommodated into an effective model of transplanckian physics including back reaction. The results, if due to a real effect, would indicate the presence of a low fundamental scale – possibly the string scale – around $2.2 \cdot 10^{-5} M_{pl}$.

June 2006

arXiv:astro-ph/0606474 v1 20 Jun 2006

1 Introduction

Recently, the third year of data from the WMAP satellite was released. The high accuracy measurements provide an opportunity not only to test inflation in general, but also to distinguish between specific models, [1]. One can even hope to obtain some important constraints on models inspired by string theory.

A particularly exciting prospect is to look for qualitatively new effects due to physics at or beyond the string or Planck scale. As have been argued in many works – the references [2-24] just represent a selected few – there are reasons to expect a characteristic signal consisting of a modulation in the primordial spectrum with a periodicity determined by the slow roll parameters. The magnitude of the effect is believed to be quite small but could nevertheless be within reach of present or upcoming observatories.

In [1] an analysis in search for such an effect was made but no significant signal was found. Intriguingly, another analysis made in [28] claim that there actually are some weak hints in the present data. According to [28], the indications have become slightly stronger in light of the WMAP3 release compared to similar earlier claims by the same authors, [29-31]. Even though the results are by no means statistically significant, it is nevertheless an interesting and useful exercise to see whether the data can be made compatible with the theoretical expectations. If nothing else, it serves as an illustration of what kind of constraints could be obtained in the event a real signal was discovered.

As noted in [28], the parameters suggested by the data correspond to a rapid oscillation with a surprisingly large amplitude. It is important to note that the claimed oscillations in amplitude are periodic in the logarithm of the scale of the CMBR fluctuations ; just as expected if due to transplanckian physics. This is related to the underlying assumption that there are new physics associated with a fixed energy scale. Other possible sources of modifications in the primordial spectrum – such as features in the inflaton potential – are typically associated with physics taking place at some definite moment in time, and give rise to other types of oscillations, [25][26][27].

Given the strength of the effect it is a valid concern whether it is at all consistent with a possible problem of back reaction on the geometry. After all, the oscillations are supposed to be caused by a nontrivial vacuum for the inflaton field that also would be expected to contribute to the energy density and change the way the universe expands. As shown in [22] and [23], however, even a large back reaction will not necessarily destroy the inflationary phase. The main purpose of this note is to address this issue in the context of the analysis made in [28]. We will make use of a formalism developed in [23] and [24] where the effect of back reaction is incorporated in a self consistent way. We will show that the back reaction is under control and fully consistent with inflation, with a slow roll found to be completely dominated by the vacuum energy given the parameters suggested by [28]. To be precise, we find evidence for a fundamental scale around $2.2 \cdot 10^{-5} M_{pl}$, where $M_{pl} \sim 2.4 \cdot 10^{18} \text{GeV}$ is the reduced Planck mass.

The outline of the paper is as follows. We begin in section two with a summary of

the phenomenological model we are using, including back reaction. We then proceed with an analysis of the WMAP3 data as given by [28], and end with some conclusions.

2 A model of transplanckian physics

The main idea, which consequences we want to explore, is that physics beyond the string or Planck scale is magnified through the expansion of the universe, and affects phenomena at lower energies such as the fluctuations of the CMBR. We will model possible new physics with a choice of vacuum different from the usual Bunch-Davies vacuum, where we assume a Bogolubov mixing linear in $\frac{H}{\Lambda}$ with the only dependence on scale being through the Hubble constant H . Λ is the energy scale of the new physics which could be the string scale or the Planck scale. We assume that all this new physics can effectively be encoded in the choice of vacuum.

As argued in [24], the effects propagating down to low energy will be of two types: a modulation of the CMBR spectrum and a back reaction on the expansion of the universe. Below we will review what the consequences are.

2.1 Effects on the CMBR

According to the analysis of [5], given a Bogolubov mixing as described above, the typical effect to be expected on the primordial spectrum is of the form

$$P(k) = \left(\frac{H}{\phi} \right)^2 \left(\frac{H}{2\pi} \right)^2 \left(1 - \frac{H}{\Lambda} \sin \left(\frac{2\Lambda}{H} \right) \right), \quad (1)$$

where we note a characteristic, relative amplitude of the correction given by $\frac{H}{\Lambda}$, and a modulation sensitively depending on how $\frac{\Lambda}{H}$ changes with k . The claim is that whatever the nature of the high energy physics really is, a modulated spectrum of this form is what we should naturally expect. In [8] one can find an early discussion of the phenomenological relevance of the effect, and how the magnitude is related to the characteristic parameters describing the inflationary phase. Using the standard slow roll approximation, where an important parameter is

$$\varepsilon = \frac{M_{pl}^2}{2} \left(\frac{V'}{V} \right)^2, \quad (2)$$

with initial conditions imposed at some fundamental scale $\Lambda = \gamma M_{pl}$, it is found that

$$\frac{\Delta k}{k} \sim \frac{\pi H}{\varepsilon \Lambda} \sim 1.3 \cdot 10^{-3} \frac{1}{\gamma \sqrt{\varepsilon}}, \quad (3)$$

and

$$\frac{H}{\Lambda} \sim 4 \cdot 10^{-4} \frac{\sqrt{\varepsilon}}{\gamma}. \quad (4)$$

These two relations are the key to estimating the expected magnitude of the effect. For instance, with a string scale a couple of order of magnitudes below the Planck scale, and a slow roll parameter $\varepsilon \sim 10^{-2}$, we find an amplitude of $\frac{H}{\Lambda} \sim 10^{-2}$ – comparable with cosmic variance – and a periodicity given by $\frac{\Delta k}{k} \sim \mathcal{O}(1)$. The data suggested by WMAP3, however, need a generalization of the analysis to be accommodated which we turn to below.

2.2 Back reaction

As explained in the introduction, the presence of a nontrivial vacuum, motivated by the presence of unknown high energy physics, raises the issue of backreaction. Focusing on the contribution to the vacuum energy coming from the non-standard vacuum, as compared with the Bunch-Davies vacuum, one finds an additional energy density naively given by $\rho_\Lambda \sim \Lambda^2 H^2$. To lowest order, as long as $\Lambda \ll M_p$, we can ignore this contribution as was concluded in [32]. In [23] and [24], however, the discussion was taken a step further and it was noted that the presence of the background energy will change the effective slow roll parameters. In fact, in our attempt to match the possible WMAP3-effect, we will find ourselves in a situation where it is the non-standard physics that dominates the slow roll of the Hubble constant.

We denote the relevant slow roll parameter by ε and define it through

$$\varepsilon = \frac{\dot{H}}{H^2}. \quad (5)$$

In addition, we still assume a rolling inflaton in the background whose main role is to end inflation. That is, it is initially governed by a slow roll parameter according to

$$\varepsilon_{\text{inf}} = \frac{\dot{\phi}^2}{2M_{\text{pl}}^2 H^2}, \quad (6)$$

where ϕ is a canonically normalized inflaton. It is ε_{inf} which, in the usual way, determines the relative amplitude between the dominating scalar modes and the tensor modes. With no back reaction from the vacuum we would have had $\varepsilon = \varepsilon_{\text{inf}}$, while we here are interested in the case where $\varepsilon_{\text{inf}} \ll \varepsilon \ll 1$, during the era when the fluctuations relevant for the CMBR are generated. Inflation ends when ε_{inf} has evolved to become of order one. As explained in [24], this leads to a decoupling of the expressions for the amplitude and the period according to

$$\frac{H}{\Lambda} \sim 4 \cdot 10^{-4} \frac{\sqrt{\varepsilon_{\text{inf}}}}{\gamma}, \quad (7)$$

and

$$\frac{\Delta k}{k} \sim \frac{\pi H}{\varepsilon \Lambda} \sim 1.3 \cdot 10^{-3} \frac{\sqrt{\varepsilon_{\text{inf}}}}{\gamma \varepsilon}. \quad (8)$$

Let us now proceed with an estimate of ε , following [23]. The above estimate of the energy density is not good enough when we want to find an expression for ε . What we need to do is to take into account that H will be changing with time, i.e. decrease. Modes with low momenta were created at earlier times when the value of H were larger, and there will be an enhancement in the way these modes contribute to the energy density. We therefore find an energy density given by

$$\rho_{\Lambda}(a) = \frac{1}{2\pi^2} \int_{\varepsilon}^{\Lambda} dp p^3 \frac{H^2\left(\frac{ap}{\Lambda}\right)}{\Lambda^2} = \frac{1}{2\pi^2} \frac{\Lambda^2}{a^4} \int_{a_i}^a da a^3 H^2(a), \quad (9)$$

where we have introduced a low energy cutoff corresponding to the energy at the time of observation of modes that started out at Λ at some arbitrary initial scale factor a_i . If we take a derivative of the energy density with respect to the scale factor and use $\frac{d}{da} = \frac{1}{aH} \frac{d}{dt}$, we find

$$\dot{\rho}_{\Lambda} + 4H\rho_{\Lambda} = \frac{1}{2\pi^2} \Lambda^2 H^3, \quad (10)$$

and we conclude that we must introduce a source term in the Friedmann equations. It was found in [23] that the evolution is governed by

$$\frac{d}{da} (a^5 H H') = -\frac{\Lambda^2}{3\pi^2 M_{pl}^2} a^3 H^2 - \frac{1}{2M_{pl}^2} \frac{d}{da} \left(a^4 (aH\phi')^2 \right), \quad (11)$$

where we let $' = \frac{d}{da}$. The first term on the right hand side is due to the presence of the non-standard vacuum, while the second term is due to the presence of the inflaton potential. In a situation where the first term dominates, we find a slow roll governed by

$$\varepsilon = \frac{\gamma^2}{12\pi^2}, \quad (12)$$

for small $\gamma = \frac{\Lambda}{M_{pl}}$.¹ In the standard case, with no vacuum contribution, the only non-vanishing term is the second one, leading to a slow roll governed by (6).

There are two ways to further generalize the model, one of which played an important role in [24]. While it is the slow roll of the inflaton that controls the overall amplitude of the primordial spectrum, one could easily imagine that there are more fields in the non-standard vacuum. These would also contribute to the back reaction and enhance ε by a factor n , where n is the number of participating fields. Another generalization was advocated in [14], where it was argued that one should allow for a free numerical factor in the Bogolubov coefficient and generalize $\frac{H}{\Lambda} \rightarrow x \frac{H}{\Lambda}$. While this makes the model less natural, it is a phenomenological parameter which observations

¹In this paper we consistently use the reduced Planck mass $M_{pl} = 1/\sqrt{8\pi G} \sim 2.4 \cdot 10^{18} \text{GeV}$. This should be kept in mind when comparing with the results in [24].

will need to tell us the value of, and it is in fact necessary in order to make sense of the hints in WMAP3. With these new parameters we find

$$\varepsilon = \frac{nx^2\gamma^2}{12\pi^2}, \quad (13)$$

and that the amplitude of the oscillation is controlled by

$$x\frac{H}{\Lambda} \sim 4 \cdot 10^{-4} x \frac{\sqrt{\varepsilon_{\text{inf}}}}{\gamma}. \quad (14)$$

3 Application to WMAP3 and conclusions

We are now ready to apply our phenomenological model to the actual data provided by WMAP3 as given in [28]. According to the analysis, the values giving the best fit to the data are found to be

$$x\frac{H}{\Lambda} \sim 0.27, \quad \varepsilon \sim 2.1 \cdot 10^{-3}, \quad \frac{\Delta k}{k} \sim 0.018, \quad \text{and } x \sim 22500. \quad (15)$$

We note that the amplitude of the claimed oscillations is quite large and implies an unnaturally large value for the dimensionless number x .² We also note that the oscillations are very rapid – tens of oscillations imposed on every acoustic peak.

Making use of the above values and the key equations (8), (13) and (14), we can now easily estimate the parameters in our model which match the data. The results we find are

$$\gamma \sim 2.2 \cdot 10^{-5} \quad (16)$$

$$\varepsilon_{\text{inf}} \sim 5.3 \cdot 10^{-11}, \quad (17)$$

where we have assumed $n = 1$; with more participating fields the values become even lower. We note that our ansatz is self consistent with $\varepsilon_{\text{inf}} \ll \varepsilon \ll 1$, and that we find a very low string scale and even lower Hubble scale.

As we have seen there are two extremes to consider when we match the data. On the one hand we have the possibility considered in, e.g. [8], where the inflationary parameters are dominated by the inflaton and the back reaction due to the vacuum energy is very small. In this case the usual relations between the slope of the spectrum, the relative magnitude of scalar and tensor modes, and the slow roll parameter ε apply. In the other extreme – relevant for the WMAP3 data – the vacuum energy dominates and the slow roll is directly linked to the fundamental scale. The inflaton, on the other hand, is almost locked and changes only very slowly. As a consequence the

²In fact, there does not seem to be any suppression of the amplitude due to the ratio between the Hubble scale and the fundamental scale.

tensor modes are much further suppressed compared with the scalar ones than what one first would expect, and we find ourselves in a situation similar to hybrid inflation. Eventually, the inflaton must enter into a regime where its potential starts to change rapidly and finally make inflation come to an end.

In this note we have argued that the effects discussed in [28], if real, can be accommodated in an effective transplanckian model with a low string scale. It is interesting to note how the model unambiguously (up to the number of fields n) constrain the fundamental scale and also tell us how the inflaton rolls even though its contribution is subdominant. While highly speculative, this exercise nevertheless shows how new observational data can be used to constrain physics relevant for string theory and quantum gravity.

Acknowledgments

The work was supported by the Swedish Research Council (VR).

References

- [1] D. N. Spergel *et al.*, “Wilkinson Microwave Anisotropy Probe (WMAP) three year results: Implications for cosmology,” arXiv:astro-ph/0603449.
- [2] R. H. Brandenberger, “Inflationary cosmology: Progress and problems,” arXiv:hep-ph/9910410. J. Martin and R. H. Brandenberger, “The trans-Planckian problem of inflationary cosmology,” Phys. Rev. D **63**, 123501 (2001) [arXiv:hep-th/0005209].
- [3] A. A. Starobinsky, “Robustness of the inflationary perturbation spectrum to trans-Planckian physics,” Pisma Zh. Eksp. Teor. Fiz. **73**, 415 (2001) [JETP Lett. **73**, 371 (2001)] [arXiv:astro-ph/0104043].
- [4] N. Kaloper, M. Kleban, A. E. Lawrence and S. Shenker, Phys. Rev. D **66** (2002) 123510 [arXiv:hep-th/0201158].
- [5] U. H. Danielsson, “A note on inflation and transplanckian physics,” Phys. Rev. D **66**, 023511 (2002) [arXiv:hep-th/0203198].
- [6] U. H. Danielsson, “Inflation, holography and the choice of vacuum in de Sitter space,” JHEP **0207**, 040 (2002) [arXiv:hep-th/0205227].
- [7] R. Easther, B. R. Greene, W. H. Kinney and G. Shiu, “Inflation as a probe of short distance physics,” Phys. Rev. D **64**, 103502 (2001) [arXiv:hep-th/0104102]. “Imprints of short distance physics on inflationary cosmology,” Phys. Rev.

- D **67** (2003) 063508 [arXiv:hep-th/0110226]. “A generic estimate of trans-Planckian modifications to the primordial power Phys. Rev. D **66** (2002) 023518 [arXiv:hep-th/0204129].
- [8] L. Bergström and U. H. Danielsson, “Can MAP and Planck map Planck physics?,” JHEP **0212** (2002) 038 [arXiv:hep-th/0211006].
 - [9] J. C. Niemeyer, R. Parentani and D. Campo, “Minimal modifications of the primordial power spectrum from an adiabatic Phys. Rev. D **66** (2002) 083510 [arXiv:hep-th/0206149].
 - [10] A. A. Starobinsky and I. I. Tkachev, “Trans-Planckian particle creation in cosmology and ultra-high energy cosmic JETP Lett. **76** (2002) 235 [Pisma Zh. Eksp. Teor. Fiz. **76** (2002) 291] [arXiv:astro-ph/0207572].
 - [11] K. Goldstein and D. A. Lowe, “Initial state effects on the cosmic microwave background and trans-planckian Phys. Rev. D **67** (2003) 063502 [arXiv:hep-th/0208167].
 - [12] U. H. Danielsson, “On the consistency of de Sitter vacua,” JHEP **0212** (2002) 025 [arXiv:hep-th/0210058].
 - [13] V. Bozza, M. Giovannini and G. Veneziano, “Cosmological perturbations from a new-physics hypersurface,” JCAP **0305** (2003) 001 [arXiv:hep-th/0302184].
 - [14] C. Armendariz-Picon and E. A. Lim, “Vacuum choices and the predictions of inflation,” JCAP **0312** (2003) 006 [arXiv:hep-th/0303103].
 - [15] J. Martin and R. Brandenberger, “On the dependence of the spectra of fluctuations in inflationary cosmology on trans-Planckian physics,” Phys. Rev. D **68** (2003) 063513 [arXiv:hep-th/0305161].
 - [16] O. Elgaroy and S. Hannestad, “Can Planck-scale physics be seen in the cosmic microwave background?,” Phys. Rev. D **68** (2003) 123513 [arXiv:astro-ph/0307011].
 - [17] T. Okamoto and E. A. Lim, “Constraining Cut-off Physics in the Cosmic Microwave Background,” Phys. Rev. D **69** (2004) 083519 [arXiv:astro-ph/0312284].
 - [18] U. H. Danielsson and M. E. Olsson, “On thermalization in de Sitter space,” JHEP **0403** (2004) 036 [arXiv:hep-th/0309163].
 - [19] K. Schalm, G. Shiu and J. P. van der Schaar, “Decoupling in an expanding universe: Boundary RG-flow affects initial JHEP **0404** (2004) 076 [arXiv:hep-th/0401164].
 - [20] J. de Boer, V. Jejjala and D. Minic, “Alpha-states in de Sitter space,” arXiv:hep-th/0406217.

- [21] R. Easther, W. H. Kinney and H. Peiris, “Observing trans-Planckian signatures in the cosmic microwave background,” JCAP **0505** (2005) 009 [arXiv:astro-ph/0412613].
- [22] R. H. Brandenberger and J. Martin, “Back-reaction and the trans-Planckian problem of inflation revisited,” arXiv:hep-th/0410223.
- [23] U. H. Danielsson, “Transplanckian energy production and slow roll inflation,” Phys. Rev. D **71** (2005) 023516 [arXiv:hep-th/0411172].
- [24] U. H. Danielsson, “Inflation as a probe of new physics,” JCAP **0603** (2006) 014 [arXiv:hep-th/0511273].
- [25] C. P. Burgess, J. M. Cline, F. Lemieux and R. Holman, JHEP **0302** (2003) 048 [arXiv:hep-th/0210233].
- [26] C. P. Burgess, J. M. Cline and R. Holman, “Effective field theories and inflation,” JCAP **0310** (2003) 004 [arXiv:hep-th/0306079].
- [27] N. Kaloper and M. Kaplinghat, “Primeval corrections to the CMB anisotropies,” Phys. Rev. D **68** (2003) 123522 [arXiv:hep-th/0307016].
- [28] J. Martin and C. Ringeval, “Inflation after WMAP3: Confronting the slow-roll and exact power spectra to CMB data,” arXiv:astro-ph/0605367.
- [29] J. Martin and C. Ringeval, “Superimposed Oscillations in the WMAP Data?,” Phys. Rev. D **69** (2004) 083515 [arXiv:astro-ph/0310382].
- [30] J. Martin and C. Ringeval, “Addendum to “Superimposed Oscillations in the WMAP Data?”,” Phys. Rev. D **69** (2004) 127303 [arXiv:astro-ph/0402609].
- [31] J. Martin and C. Ringeval, “Exploring the superimposed oscillations parameter space,” JCAP **0501** (2005) 007 [arXiv:hep-ph/0405249].
- [32] T. Tanaka, “A comment on trans-Planckian physics in inflationary universe,” arXiv:astro-ph/0012431.